

# Effective field theory at moderate finite density

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Effective field theory of the in-medium nucleon-nucleon interaction is considered. The effective range parameters are found to be of a natural scale. The low density limit is discussed both in perturbative and nonperturbative situations. In the nonperturbative case the attractive character of the nucleon-nucleon interactions in the  $^1S_0$  channel leads to the nuclear superfluidity which is analyzed in the framework of the renormalization group. The numerical values of the corresponding energy gap are in agreement with results obtained by more traditional many-body techniques. The S-wave part of potential energy per particle is calculated for different values of nuclear density. The role of pion and many-body effects is discussed. Problems and challenges in constructing the chiral theory of nuclear matter are outlined.

## 1. Introduction.

Recently, there has been much interest in applying the Effective Field Theory (EFT) [1] methods to study nuclear interactions. The key element of EFT is the counting rules allowing for a consistent expansion scheme with predictable theoretical errors. Another important ingredient of any EFT is a separation of scales so that the high frequency modes can be integrated out and manifest itself indirectly in the Low-Energy-Effective Constants (LEC's). In addition, the symmetries of the underlying “microscopic” theory put important constraints on the interaction terms in the effective Lagrangian, making them less arbitrary as compared to the sometimes more traditional phenomenological description. In the context of nuclear matter such a description implies the use of some phenomenological potentials leading, in general, to the different effective in-medium interactions and bringing uncontrollable uncertainties, related to the off-shell ambiguities. These ambiguities can, at least in principle, be eliminated in the EFT approach by using the freedom to work in the different low-energy representations of the underlying QCD. In the other words EFT may be useful to get rid of the long-standing problem arising in the more standard treatments of hadron interactions and related to the necessity of utilizing of the phenomenological form-factors to regulate the divergent integrals and parameterize the complicated internal structure of hadrons. There is nothing wrong to use them in the calculations based on phenomenological models. However, the problem is that the underlying field theory must then be nonlocal making the whole issue of constructing the entirely consistent field-theoretical treatment of the low-energy hadron interactions extremely difficult to deal with. Moreover, when the problem under consideration is the interaction of hadrons with the external electromagnetic fields, the straightforward use of form-factors may lead to the conflict with gauge invariance. All these features make the reanalysis of nuclear matter in the framework of EFT useful and promising. The main element of nuclear dynamics is the elementary nucleon-nucleon interactions. Because of the large scale involved in this process the EFT treatment of the NN-interactions in vacuum must be nonperturbative. In the framework of EFT the problem was first considered by Weinberg [2] who suggested to apply chiral counting rules to the set of all possible irreducible diagrams at given order which are then to be iterated in all order by solving Lippmann-Schwinger equation. In somewhat different approach proposed by Kaplan, Savage and Wise (KSW) [3] only certain subclass of lowest order diagrams was summed up to all orders whereas the rest was treated as a perturbation so that KSW counting is applied directly to the scattering amplitude. The analysis [19] of the NN (pionless!) scattering amplitude based on the Wilson renormalization group has demonstrated that power counting indeed closely follows that which was found by KSW.

The extension of these developments to the case of finite density is by no means straightforward [5]. There are several points which must be taken into consideration to formulate the consistent EFT at finite baryon density. First, being unnaturally large in vacuum the NN S-wave scattering lengths get significantly reduced in nuclear matter due to Pauli principle and many-body interactions. Second, the way of formulating the counting rules allowing for the consistent treatment of the many-body effects, such as the ring and ladder diagrams must be found. Third, the long-range part of the NN effective interaction related to the pion exchange in nuclear environment should be taken into account. In addition, the three and four nucleon interaction terms must be included in the EFT Lagrangian.

Some recent studies of the systems with the finite fermion density in the context of EFT have been focused on constructing of the chiral expansion for the dilute interacting Fermi-gas [6] and mean-field effective chiral Lagrangian [7]. The issues of the in-medium counting rules [8] and nuclear binding in chiral limit [9] were also examined recently. In this paper we focus on the EFT approach to construct the analog of the Brueckner G-matrix and try to relate the in-medium LEC's and those obtained in the free space fit. It seems to be quite a nontrivial problem to establish such relations since the LEC's entering the effective potential of the in-medium NN interactions may be density dependent.

Any EFT expansion has a form in  $k/\Lambda$ , the ratio of some generic momentum involved and a characteristic short range physics mass scale. So it is important to establish a relevant expansion parameter. The analysis of the different phenomenological models indicates that the nonrelativistic mean field approximations, successfully describing the bulk nuclear properties at low and moderate energy scale, encounter difficulties when applied to the nuclear phenomena with a characteristic mass scale around 600 MeV (electromagnetic form-factors at relatively large momentum transfer, for instance). This is the region where the effect of the short range correlations (SRC) becomes important. Thus, it is plausible that the typical scale where the EFT description of nuclear matter with pointlike nucleon-nucleon interaction can be close to the inverse range of the SRC. One notes that the addition of the long-ranged pion effects should not significantly change this estimate. As the typical nuclear momentum scale is of order the Fermi-momentum  $p_F \simeq 260\text{-}270$  MeV the relevant expansion parameter is close to 0.3 - 0.4. This admittedly crude estimate can be improved if the influence of the nuclear many-body environment is taken into account.

All essential features of the nucleon-nucleon interactions can, in principle, be obtained from the effective chiral Lagrangian

$$\mathcal{L} = N^\dagger i \partial_t N - N^\dagger \frac{\nabla^2}{2M} N - \frac{1}{2} C_0 (N^\dagger N)^2 - \frac{1}{2} C_2 (N^\dagger \nabla^2 N) (N^\dagger N) + h.c. + \quad (1)$$

We consider the simplest case of the NN scattering in the  ${}^1S_0$  state and assume zero total 3-momentum of NN pair in the medium. The inclusion of the nonzero total 3-momentum does not really change anything qualitatively and only makes the calculations technically more involved. The G-matrix, describing the dynamics of the in-medium interactions satisfies the BG equation

The G-matrix is given by

$$G(p', p) = V(p', p) + M \int \frac{dq q^2}{2\pi^2} V(p', q) \frac{\theta(q - p_F)}{M(\epsilon_1(p) + \epsilon_2(p')) - q^2} G(q, p), \quad (2)$$

Here  $\epsilon_1$  and  $\epsilon_2$  are the single-particle energies of the bound nucleons. In nuclear medium bound nucleon acquires the effective mass slightly different from that in free space. In a fully self-consistent calculations the nuclear mean field and chiral G-matrix should, in principle, be treated using the same Lagrangian. However, it requires the inclusion both pion and many-body effects and establishing the power counting rules at finite density. In this paper we explore an easier approach and use the in-medium value of the nucleon mass close to the accepted one in the phenomenological nonrelativistic [10] mean field treatments. We used the value  $M = 0.75M_0$  in the calculations at  $p_F = 1.36 \text{ fm}^{-1}$ , where we denote  $M_0$  the nucleon mass in vacuum. In order to check the sensitivity of the results obtained to the different choices of effective nucleon mass the values of  $M$  have been varied from  $0.7M_0$  to  $0.8M_0$ . The higher order changes of the results were found for the potential energy per particle so we will keep the value of  $M$  fixed and discuss the results corresponding the choice  $M = 0.75M_0$ .

This paper is organized as follows. In the next section we consider an exactly solvable model to estimate the in-medium effective range parameters. Then the generalization of the effective nucleon-nucleon Lagrangian to the finite density case is discussed. In the section 4 we consider the low-density limit in the case when the perturbative treatment is possible. The issue of superfluid nuclear gap is discussed in the section 5. Section 6 offers a brief summary of results for a potential energy of nuclear matter and some concluding remarks.

## 2. Exactly solvable model.

In order to get an idea about the typical size of the in-medium effective range parameters which, in turn, determine the typical scale of the problem it seems reasonable to exploit some exactly solvable model with the parameters adjusted to describe the experimental data or empirical results. Moreover, the in-medium scattering amplitude obtained in such solvable model can further be used to extract the values of the effective couplings in the corresponding EFT.

In some sense this is similar to the procedure usually adopted in the vacuum case. Firstly, one should determine the LEC's from, say, phase shifts and then use these LEC's to calculate the other observables. Unfortunately, there is no such thing as phase shift analysis in nuclear matter. One could instead rely on some solvable model with the parameters adjusted to describe nuclear data to extract the numerical values of the effective couplings. However, there is an important difference between vacuum and in-medium cases. In the former the corresponding LEC's are extracted from the model independent experimental data whereas in the latter the in-medium LEC's are determined from the model dependent scattering amplitude which may, in general, differ for the phase-equivalent NN potentials. In this respect it would be desirable to relate the vacuum and in-medium effective couplings to be able to use the experimental data on the NN scattering in vacuum. We will discuss this and related issues in more details below.

We use a model with a simple separable interaction

$$V = -\lambda|\eta\rangle\langle\eta| \quad (3)$$

with the form factors

$$\eta(p) = \frac{1}{(p^2 + \beta^2)^{1/2}} \quad (4)$$

One can easily get

$$\frac{1}{T(k, k)} = V(k, k)^{-1} \left[ 1 - M_0 \int \frac{dq q^2}{4\pi^2} \frac{V(q, q)}{k^2 - q^2} \right] \quad (5)$$

The analytic solution for the corresponding  $T$ -matrix is straightforward

$$\frac{1}{T(k, k)} = V(k, k)^{-1} \left[ 1 - M_0 \int \frac{dq q^2}{4\pi^2} \frac{V(q, q)}{k^2 - q^2} \right] \quad (6)$$

The G-matrix being the solution of the BG equation for the separable interaction takes the form

$$G(k, k) = -\eta^2(k) \left[ \lambda^{-1} + \frac{M}{2\pi^2} \int dq q^2 \frac{\theta(q - p_F)\eta^2(q)}{k^2 - q^2} \right]^{-1} \quad (7)$$

The empirical value of the potential energy per particle is  $\sim -16$  MeV and can be reproduced with

$$\lambda = 1.95 \quad \beta = 0.8 \text{ fm} \quad (8)$$

These parameters being substituted to the G-matrix in the low-momentum limit  $G(k \rightarrow 0)$  lead to the estimates  $a_m \simeq r_m \simeq 0(1)$ , where  $a_m$  and  $r_m$

are the in-medium analogs of scattering length and effective radius. So one can conclude that the in-medium effective range parameters are of a natural size which is roughly given by the value of the Fermi-momentum at nuclear saturation  $p_F \simeq 1.36 \text{ fm}^{-1}$ . This allows us to avoid some part of the difficulties typical for the EFT for the nucleon-nucleon interaction in vacuum such as necessity of a special treatment for the system with the unnaturally large scattering length. Of course, there are many other specific points typical for the finite density EFT and making its consistent yet practical realization quite nontrivial. Some of them will be considered in what follows. As already mentioned above the in-medium phenomenological amplitude may depend on the model of the NN interaction used. To check whether the freedom to choose among phenomenologically equivalent models of the NN interactions may give rise to the significant changes of our estimates of the in-medium effective range parameters we carried out the calculations with the exponential parameterization of the form-factors entering the separable potential. The values of  $a_m$  and  $r_m$  found are as well of a natural size  $\simeq 0(1)$  and its numerical values are close to those obtained with the “square root” fit. So it seems that the conclusion about a natural size of the the in-medium effective range parameters is quite robust and practically model independent. Physically it looks quite natural. Firstly, the integrand in the BG equation does not have a pole irrespectively of the explicit form of the kernel. Secondly, the strength of the shallow virtual nucleon-nucleon bound state gets significantly reduced in nuclear medium because of interaction with nuclear mean field leading to the inevitable decrease of the scattering length.

### 3. Effective Lagrangian treatment.

The solution of the BG equation with the NN potential extracted from the effective Lagrangian is similar to that in the vacuum case [11] and given by

$$\frac{1}{G(k, p_F)} = \frac{(C_2 I_3(p_F) - 1)^2}{C_0 + C_2^2 I_5(p_F) + k^2 C_2 (2 - C_2 I_3(p_F))} - I(k, p_F), \quad (9)$$

where we the loop integrals are

$$I_n(p_F) \equiv -\frac{M}{(2\pi)^2} \int dq q^{n-1} \theta(q - p_F). \quad (10)$$

and

$$I(k, p_F) \equiv \frac{M}{2\pi^2} \int dq \frac{q^2 \theta(q - p_F)}{k^2 - q^2}. \quad (11)$$

These loop integrals are divergent and therefore the procedure of regularization and renormalization must be carried out. Note that the issue of the

nonperturbative renormalization is quite a subtle problem. In contrast to the standard perturbative case where the usual field theoretical methods can be used to regularize the given divergent graphs and then renormalize the bare coupling constants, in the nonperturbative situation the renormalization of the whole integral equation must be carried out. In the case when the analytic solution for the scattering amplitude can be obtained as, for example, in the pionless nucleon-nucleon EFT, the renormalization of the amplitude is a rather straightforward procedure. However, if the explicit solution is not possible (this is the case for the realistic NN forces) then the special care is needed to perform the renormalization in a consistent way [12]. In this paper we follow the procedure used in Ref. [13] to renormalize the effective NN amplitude in vacuum. We subtract the divergent integrals at some kinematical point  $p^2 = -\mu^2$ . After subtraction the renormalized G-matrix takes the form

$$\frac{1}{G(k, p_F)} = \frac{1}{C_0(\mu, p_F) + 2k^2 C_2(\mu, p_F)} + \frac{M}{4\pi} \left[ k \ln \frac{p_F - k}{p_F + k} - i\mu \ln \frac{p_F - i\mu}{p_F + i\mu} \right], \quad (12)$$

where the couplings  $C_{0(2)}$  should now be interpreted as the renormalized quantities depending on some renormalization scale  $\mu$ . It is easy to see that in the  $p_F \rightarrow 0$  limit the vacuum chiral NN amplitude is recovered. The  $\mu$  dependence of LEC's is governed by the renormalization group (RG) equations. We require G-matrix to be a subtraction point independent quantity. Applying  $\partial/\partial\mu$  to the expression for the G-matrix and setting  $\partial G/\partial\mu = 0$  one can get the following RG equations

$$\frac{\partial C_0(\mu, p_F)}{\partial \mu} = \frac{C_0^2 M}{2\pi^2} \left( \frac{\mu p_F}{p_F^2 + \mu^2} + \arctan\left(\frac{\mu}{p_F}\right) \right) \quad (13)$$

$$\frac{\partial C_2(\mu, p_F)}{\partial \mu} = \frac{C_0 C_2^2 M}{\pi^2} \left( \frac{\mu p_F}{p_F^2 + \mu^2} + \arctan\left(\frac{\mu}{p_F}\right) \right). \quad (14)$$

In the limit  $p_F \rightarrow 0$  these equations transform to the ones derived by Kaplan et al. [3]. The solutions of these RG equations are

$$C_0(\mu, p_F) = \frac{C_0(\mu_0, p_F)}{1 + \frac{M}{2\pi^2} (\mu_0 \arctan(\frac{\mu_0}{p_F}) - \mu \arctan(\frac{\mu}{p_F})) C_0(\mu_0, p_F)} \quad (15)$$

and

$$C_2(\mu, p_F) = C_2(\mu_0, p_F) \left[ \frac{C_0(\mu, p_F)}{C_0(\mu_0, p_F)} \right]^2 \quad (16)$$

In order to determine the scale dependence of the effective couplings one needs to define its boundary values at some kinematical points. To extract these values we equate the EFT and phenomenological expressions for the G-matrix

at  $p = p_F/2$  and  $p = p_F/3$ . This is somewhat similar to the procedure used to get the values of the vacuum LEC's when the fit is usually done using the experimental phase shifts within some kinematical region. One notes that when  $p_F \rightarrow 0$  the above expressions for the effective couplings reduce to these obtained in Ref. [3] The assumed value of the Fermi-momentum is  $p_F = 1.36\text{fm}^{-1}$ . If the value  $\mu = 0$  is chosen as a subtracting point we find  $C_0(\mu = 0, p_F) = -1.88\text{fm}^2$  in LO. For the lowest order coupling fixed at NLO we get  $C_0(\mu = 0, p_F) = -2.67\text{fm}^2$  and  $C_2(\mu = 0, p_F) = 0.85\text{fm}^4$ . Substituting these values in the RG equations we get  $C_0(\mu = m_\pi, p_F) = 2.35\text{fm}^2$  and  $C_2(\mu = m_\pi, p_F) = 0.64\text{fm}^4$ . The obtained effective coupling  $C_0$  at  $\mu = m_\pi$  is fairly close to its vacuum value [3]. Thus one can conclude that the density dependence of the in-medium LO couplings is rather moderate, provided that  $\mu \simeq p_F$ . It means that at densities smaller than the normal nuclear one there is a possibility to use the vacuum values of the corresponding LEC's to get an idea about the approximate order of their in-medium analogs. It seems that such a matching of the vacuum and in-medium LEC's could provide a quite important additional possibility allowing one to put some constraints on the values of the in-medium LEC's since the vast experimental data on the NN scattering in vacuum can be used in the calculations at finite (although moderate) densities. It may also turn out useful when calculating the nuclear saturation curve since it helps to avoid the procedure of fixing the phenomenological in-medium amplitudes at different nuclear densities which would otherwise be needed to fix the density dependence of the effective couplings, making the whole EFT approach less predictable and reliable. Note that, although this matching can be carried out at any value of  $\mu$  the choice  $\mu = 0$  is rather inconvenient one since the values of the vacuum and in-medium LEC's compared at  $\mu = 0$  are completely different so that the conclusion about the moderate density dependence of the effective couplings may no longer be true at  $\mu = 0$ . Thus, the choice  $\mu \simeq m_\pi \simeq p_F$  looks more natural. We emphasize that the observables, expressed via G-matrix do not depend on the value of  $\mu$  chosen. However, it is much more difficult to relate the in-medium and vacuum NN-forces when the value  $\mu = 0$  is taken for the subtraction point.

We observe approximately 35% change in the value of  $C_0(\mu = m_\pi, p_F)$  when going from LO to NLO. It indicates that the chiral expansion is systematic in a sense that the NLO corrections lead to the “NLO changes” of the effective couplings already determined at LO. The natural size of the in-medium effective range parameters, moderate changes experienced by the LO coupling constant  $C_0$  and smallness of the NLO LEC's might, in principle, indicate the possibility of the perturbative calculations. However, in spite of this, it is still more useful to treat this problem in the nonperturbative manner. There are few reasons for this. Firstly, the overall (although distant)

goal of the EFT description is to derive both nuclear matter and the vacuum NN amplitude from the same Lagrangian. However, it is hard to say at what densities the dynamics becomes intrinsically nonperturbative, so it is better to treat the problem nonperturbatively from the beginning. The nonperturbative treatment may also turn out important to get the correct saturation curve since at some density lower than the normal nuclear one the scattering length starts departing from its natural value and some sort of the nonperturbative approach becomes inevitable. Besides, the logarithmic terms in the chiral G-matrix are important to ensure correct threshold behavior of the in-medium NN-amplitude. Secondly, the NLO corrections themselves are quite significant. Thirdly, the nonperturbative treatment is required to study the phenomena of nuclear superfluidity [14] when the perturbation theory is not valid. Finally, in the processes involving both the nonzero density and temperature, such as heavy ion collisions, the value of the Fermi-momentum can effectively be lowered again making the nonperturbative treatment preferable.

As we mentioned in the introduction , one of the most important (and yet unsolved) problems of constructing EFT at finite density is the formulation of the corresponding counting rules. The complete solution of this problem is possible only if pion effects and many-body forces are taken into account in a chirally invariant manner. However, as a first step in this direction one could formulate the “naive” counting rules for the LEC’s  $C_{0(2)}$  used in the effective Lagrangian. Since the main reason of the anomalous KSW counting  $C_{2n} \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}}$  suggested by Kaplan et al. [3] is no longer the case in nuclear medium it would be natural to count like  $C_{2n} \sim \frac{4\pi}{M\Lambda^{2n+1}}$ . However, one could modify this counting taking into account a further small scale  $Q \sim p_F \sim m_\pi \sim \mu$  involved at nonzero density so that taking  $p_F \rightarrow 0$  limit gives rise to the KSW scheme. So we count

$$C_{2n} \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}} \left[ \frac{\mu + p_F}{-\mu - (p_F/\mu)(\Lambda - \mu)} \right]^{n+1}. \quad (17)$$

Assuming  $\mu \sim p_F$  one indeed observes the moderate dependence of the LEC’s on density. The above stated values of LEC’s are consistent with this power counting. Note that in order to be fully consistent the density dependent nucleon mass should, in principle, be used in Eq.(17). However, to get the crude estimate of the effective couplings the use of vacuum value of nucleon mass is sufficient. It is clear that the exact form of the density dependence of the effective couplings is not unique. Any functional form providing the smooth interpolation between the “natural” counting  $C_{2n} \sim \frac{4\pi}{M\Lambda^{2n+1}}$  at normal nuclear density and the KSW counting  $C_{2n} \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}}$  at zero density is acceptable. It is plausible however that all forms will give the same order-of-magnitude estimate of the effective coupling. When calculating some observables at normal

nuclear density the simplest thing is to make use of the “natural counting”. The further understanding of the density dependence of the in-medium LEC’s could be achieved considering the nuclear saturation since the saturation curve would be quite sensitive to the exact values of the LEC’s at different densities. However, to treat the saturation in a consistent manner the higher partial waves (at least P and D waves) and pion effects must be included. We relegate the discussion of nuclear saturation to the separate paper. Note that although the value of the in-medium cutoff parameter  $\Lambda$  may also differ from its vacuum value we do not expect this difference to be significant. It seems to be rather unlikely that the introduction of the additional low-momentum scale  $p_F$  can lead to the large changes of the cutoff  $\Lambda$ , which is meant to be the indirect manifestation of the truly short-range physics which is only moderately touched upon by the medium effects.

Assuming that the values of the effective couplings are fixed at zero nuclear density and using the above written expression for the couplings as the functions of density one can determine the LEC’s at any intermediate value of the Fermi-momentum.

Now the remark, concerning the naturalness criteria [15] is in order. The naturalness concept in the context of nuclear interactions was elaborated in [7]. According to this concept of an individual term in the effective Lagrangian can schematically be written as some dimensionless factor of order unity multiplied by the certain combination of scale factors, characterizing the scales involved

$$C \sim c \left[ \frac{\psi^+ \psi}{f_\pi^2 \Lambda} \right]^l \left[ \frac{\partial}{\Lambda} \right]^n (f_\pi \Lambda)^2. \quad (18)$$

Applying the scaling rules developed in [7] to extract all scale factors and assuming that the cutoff parameter  $\Lambda \simeq 500$  MeV one finds  $c_0(c_2) \simeq 0(1)$ . Thus the dimensionless coefficients are indeed compatible with naturalness.

#### 4. Low-density limit. Perturbative case

The low density limit has often served as a somewhat toy many-body problem, useful for understanding the structure of the theory and checking the consistency of the approximations made. In this section we consider the case when all interactions are natural so that the perturbative treatment is valid. The corresponding expression for the potential energy per particle of the dilute Fermi-gas is well known and was derived by traditional many-body methods long time ago. We show in this section that in the limit when the perturbative expansion is valid ( $G \sim V$ ) the LO term of the low-density expansion can be reconstructed within our approach. Note that the low-density expansion including both leading and higher order terms has recently been rederived in ref. [6] in the framework of perturbative EFT. The pairing phenomena requiring the nonperturbative treatment will be considered in the next section.

The potential energy of the system of the interacting fermions is

$$U_{tot} = \frac{1}{2} \sum_{\mu,\nu} < \mu\nu | G(\epsilon_\mu + \epsilon_\nu) | \mu\nu - \nu\mu > \quad (19)$$

The summation goes over the states with momenta below  $p_F$ . Since by assumption the perturbative expansion is valid we put ( $G \sim V \sim C_0$ ). The explicit expression for the  $S$ -wave part of ground-state energy per particle is

$$\begin{aligned} \frac{E}{A} = & \frac{3p_F^2}{10M_0} + \frac{3}{\pi^2 p_F} (2T+1) \int_0^{p_F} dk k^2 G(k, p_F) \\ & \left[ \int_0^{p_F - k} dP P^2 + \int_{p_F - p}^{(p_F - p)^{1/2}} dP P^2 \frac{p_F - P^2 - k^2}{2Pk} \right], \end{aligned} \quad (20)$$

where  $P$  is the total momentum of the pair. Since we consider the low-density limit it is legitimate to use the relation  $C_0 = \frac{4\pi a}{M_0}$  between the effective coupling  $C_0$  and scattering length  $a$ . After the momentum integration is done we obtain the following lowest order expression for the ratio  $\frac{E}{A}$  in the case of the dilute Fermi-system

$$\frac{E}{A} = \frac{p_F^2}{M_0} \left[ \frac{3}{10} + \frac{1}{3\pi} (p_F a) + O(p_F^2) \right], \quad (21)$$

which coincides with the well known textbook expression [16]. The next analytic in  $p_F$  terms of the low-density expansion can be derived if NLO term with the effective coupling  $C_2$  is included. The result also agrees with that obtained by standard many-body technique. One notes that it may be nontrivial to go further in the low-density expansion since some higher order terms appear in the conventional expression multiplied by powers of  $\ln(p_F a)$ . As argued in Ref. [6] in order to reproduce these terms in the framework of the perturbative EFT the graphs describing three-to-three scattering amplitude must be included. The point is that the diagram for the two-to-two scattering contain only power divergences and thus, being regularized by means of dimensional regularization with minimal subtraction, can only lead to the terms which are analytic in  $p_F/\Lambda$ . Note that it may not be so in the nonperturbative situation with unnatural scattering length since the G-matrix from Eq.(12) already contains the logarithms of  $p_F$  needed to reproduce the terms with  $\ln(p_F a)$  in the low-density expansion so that no additional 3-body terms are needed. Of course, in the case of large scattering length the low-density expansion has a little practical use since it is valid for very low values of  $p_F$ , satisfying the condition  $p_F a < 1$

## 5. Low-density limit. Nonperturbative case

It is well known that nuclear matter is superfluid. The nucleon-nucleon interactions is attractive in the  $^1S_0$  channel so that the corresponding amplitude develops the singularity, no matter how weak the interaction is. This singularity is cured by the formation of Cooper pairs. The superfluid properties of nuclear matter are important in the study of neutron stars [17] and heavy nuclei close to the drip line [18]. The studies of nuclear superfluidity have basically been carried out using pairing matrix element given by the bare nucleon-nucleon interactions as the elementary input entering the many-body calculations so it would be useful to see how nuclear superfluidity is described when the basic nucleon-nucleon interaction is treated in the framework of EFT.

The gap exhibited by the system is formed at the Fermi surface. Since Cooper pair condensation cannot be described by means of a perturbation theory a nonperturbative treatment is required. The standard approaches to the problem of nuclear superfluidity are based on the traditional many-body methods such as coupled-cluster theory [19] or self-consistent Green function theory [20]. To our knowledge the first attempts to analyze the nuclear superfluidity in the framework of EFT was made in Refs. [21,22]. The discussion below follows the line which is somewhat similar to that in Ref. [21]. The aim of this section is to use the RG equations to estimate the size of the superfluid gap at small nuclear densities. In the RG technique the size of the gap is determined by the position of the singularity in a running effective coupling near the Fermi surface [23]. Let's define

$$\frac{\mu_0}{p_F} \arctan\left(\frac{\mu_0}{p_F}\right) = t_0, \quad \frac{\mu}{p_F} \arctan\left(\frac{\mu}{p_F}\right) = t. \quad (22)$$

After substitution to Eq.(16) one gets

$$C_0(t, p_F) = \frac{C_0(t_0, p_F) 2\pi^2}{2\pi^2 + M p_F (t_0 - t) C_0(t_0, p_F)}. \quad (23)$$

We take the limit  $p_F \rightarrow 0$  so that the effective coupling  $C_0$  extracted from the nucleon-nucleon scattering in vacuum can be used

$$C_0(\mu_0, p_F \rightarrow 0) \simeq C_0(\mu_0) = \frac{4\pi}{M_0} \left( \frac{1}{-\mu_0 + 1/a} \right). \quad (24)$$

We denote  $a$  the vacuum scattering length. One can see that the coupling  $C_0$  has singularity at

$$\nu^* = \frac{4\pi}{M_0 C_0(t)} + \frac{2p_F}{\pi} t. \quad (25)$$

The pairing gap is expected to be of size  $\sim \exp(-\frac{\pi}{2p_F}\nu^*)$ . Assuming  $t = 0$  one can rewrite the expression for the gap in terms of the scattering length

$$\Delta \sim b \exp\left(\frac{\pi}{2p_F a}\right). \quad (26)$$

This expression coincides with one obtained in Ref. [22]. However, as pointed out in [3] the choice  $\mu = 0$  is not the optimal one. The choice  $\mu \sim p$  makes the power counting more transparent. Moreover, at  $\mu \sim 0$  the density dependence of the  $C_0$  may become nonnegligible even at low densities so the assumption  $C_0(\mu, p_F) \simeq C_0(\mu, p_F = 0)$  is better justified when  $\mu > p_F$ . The scattering amplitude is independent of the subtraction point so the choice of this point is just a matter of practical convenience. We used the value  $C_0(\mu) = -2.23 fm^2$  determined at  $\mu = 1.2 fm^{-1}$  and calculated the gap  $\Delta(p_F)$  as a function of Fermi momentum. One notes that the RG analysis gives the order of magnitude estimate of the gap but cannot provide the determination of the preexponential factor which may be important at small  $p_F$ . This factor can be found by matching to the known results, obtained at low density [16]. The result of this matching is

$$b \simeq \frac{p_F^2}{2M}. \quad (27)$$

The results of calculations of  $\Delta(p_F)$  are

$$\begin{aligned} \Delta(p_F = 0.1 fm^{-1}) &= 0.075 MeV, \Delta(p_F = 0.2 fm^{-1}) = 0.3 MeV, \\ \Delta(p_F = 0.4 fm^{-1}) &= 0.76 MeV, \Delta(p_F = 0.6 fm^{-1}) = 2.39 MeV, \end{aligned} \quad (28)$$

We stopped calculations of the gap at  $p_F = 0.6 fm^{-1}$  since at higher values of Fermi momentum the condition  $\mu > p_F$  is no longer valid and the density dependence of the effective couplings may affect the results of calculations.

Our results are in a qualitative agreement with the calculations using more traditional approaches. However, as well known from the phenomenological studies at  $p_F \simeq 1.7 fm^{-1}$  the gap vanishes. The physical reason is that at such densities the attraction in the  ${}^1S_0$  channel is replaced by the repulsion so that the formation of the fermion condensate is no longer possible. The EFT treatment must be modified to incorporate the effect of the decrease of a superfluid gap with increasing density since the ration  $p_F/\Lambda$  is not a reliable expansion parameter any longer and density dependence of the LEC's should be taken into account. The problem of the EFT description of nuclear superfluid gap in the wider range of densities will be addressed in a separate paper.

## 6. Nuclear matter observables

Let's now discuss the results of calculations of potential energy of nuclear matter. As in the low-density case the Eq.(20) is used. Let's start from the results of calculations at normal nuclear density  $\rho_0 = 1.36 fm^{-3}$ . In the

LO perturbative calculation assuming  $G \sim C_0$  one gets  $\frac{U(1S_0)}{A} \simeq -11.9 MeV$ . Making use of the nonperturbative expression for the G - matrix but keeping only LO in the chiral expansion of the effective nucleon-nucleon potential gives rise to the value  $\frac{U(1S_0)}{A} \simeq -17.0 MeV$ . The inclusion of the NLO terms in the chiral Lagrangian leads to  $\frac{U(1S_0)}{A} \simeq -13.0 MeV$ . So the corresponding correction is about 30% and can be attributed to NLO. One notes that both  $\frac{U(1S_0)}{A}$  and LO effective coupling  $C_0$  experience NLO corrections when NLO terms are included in the effective Lagrangian. It justifies one more time the statement about the consistency of the corresponding chiral expansion. The similar calculations performed in the triplet S-wave channel leads to the value  $\frac{U(3S_1)}{A} \simeq -17.3(-13.2) MeV$  in LO (NLO). However, there is an important difference between the singlet and triplet channels because of the S-D mixing arising at NLO in the triplet channel. This effect is not considered in this paper since it requires the inclusion of the explicit pion degrees of freedom in the Lagrangian. The issue of the pionic effects is discussed in more details below. Here we only note that the pion effects must eventually be included in a chirally symmetric manner consistent with the in-medium counting rules that are yet to be established. It makes the whole issue of the pionic effects quite complicated. To calculate the value of  $\frac{U}{A}$  at lower densities we use Eq.(17) to fix the effective couplings  $C_0$  and  $C_2$ . We use the value  $\Lambda = 2.05 fm^{-1}$  for the effective cutoff parameter. The effective couplings determined at different densities and at  $\mu = m_\pi$  are

$$\begin{aligned} C_0(\mu, p_F = 1.2 fm^{-1}) &= -2.41 fm^2; C_2(\mu, p_F = 1.2 fm^{-1}) = 0.98 fm^4 \\ C_0(\mu, p_F = 1.0 fm^{-1}) &= -2.48 fm^2; C_2(\mu, p_F = 1.0 fm^{-1}) = 1.05 fm^4 \\ C_0(\mu, p_F = 0.8 fm^{-1}) &= -2.53 fm^2; C_2(\mu, p_F = 0.8 fm^{-1}) = 1.16 fm^4 \end{aligned} \quad (29)$$

For the potential energy per particle we find  $\frac{U(p_F)}{A} = -11.5 MeV$  (-9.7 MeV, -8.1 MeV) at  $p_F = 1.2 fm^{-1}$  ( $1.0 fm^{-1}$ ,  $0.8 fm^{-1}$ ) respectively. These results look quite reasonable, although somewhat smaller than the values obtained in more traditional approaches. We stopped the calculations at  $p_F = 1.36 fm^{-1}$  which is rather close to the nuclear saturation point. The approach should be modified in the several aspects in order to be applied to the nuclear systems with the density higher than the normal nuclear one. First, the Fermi-momentum, being only “marginally light scale” already at normal nuclear density becomes closer to the generic “heavy” scale. Consequently, the counting rules must be modified and the value of the effective cutoff increased. At some critical density the value of cutoff can reach the point, where the other degrees of freedom such as vector mesons and  $\Delta$ -isobars are to be included explicitly. It brings in the other sources of uncertainties which are rather difficult to control. Moreover, whereas at normal nuclear density and typical energy scale  $Q \sim 100$

MeV the relativistic corrections enter at higher orders, at some critical density the whole formalism should be formulated in a fully relativistic way. It considerably complicates the practical use of EFT. The counting rules must be significantly modified to take into account the excitations both the Fermi and Dirac seas and the fact that the nucleon mass can no longer be considered as a genuine “heavy” scale. On the other hand, at densities much higher than the normal nuclear one the matter was shown to be a color superconductor [24] where quark and gluon degrees of freedom must be considered explicitly. When the density is lowered a color superconducting phase should match the EFT, formulated in terms of baryons and mesons. Carrying out this matching is a highly nontrivial problem, requiring at least qualitative understanding of the mechanism of hadronization at high densities [25]. Therefore some model of how confinement occurs at high densities is probably needed.

As already mentioned above, to be completely consistent, many-body forces and pion effects must be included in the EFT treatment. The contribution from the many-body forces is expected to be rather small. There are at least two reasons for such an expectation. First, each additional interacting nucleon line will bring in the additional small factor  $Q^3 \sim p_F^3$  in the expression for the binding energy of nuclear matter. Second, following the line of arguments suggested in [26] the contributions of the N-nucleon forces can schematically be written as  $V_N \sim Q^N / \Lambda^{N-1}$  where  $\Lambda$  is the generic large-mass QCD scale close to 1 GeV and  $Q$  is the typical low-energy scale taken to be about 0.1 GeV. Using these numbers one can see that the contribution of the three-body forces is order of magnitude smaller compared to the two body ones and that four-body interaction can safely be neglected. We stress, however, that taking into account the 3-nucleon forces may turn out important to get the saturation point close to the empirical one. The expected smallness of the many-body forces compared to the two-body ones is also confirmed by the results of several other studies both for nuclear matter and for finite nuclei with  $A > 3$  [27].

Pion degrees of freedom, being another important ingredient of any nuclear EFT, seem to average to NLO effects, when the S-wave part of nuclear matter binding energy is being calculated. One notes that this conclusion is partly supported by the results, obtained in the phenomenologically very successful Skyrme approach [28,29]. The Skyrme interaction has the pointlike form and does not contain the explicit pions. In some sense the Skyrme interaction can be considered as a parameterization of the G-matrix [30], written in the form closed to that we used. Furthermore, as shown in [31] adding explicit pions can lead only to the moderate changes of the effective couplings. This fact can qualitatively be explained by the partial cancelation from the pointlike interaction and one-pion-exchange interaction. The existence of such a cancelation was also pointed out in [8]. However, it is worth noting that the relative con-

tribution due the pionic effects may depend on the partial channel considered. For example, we expect that the pion degrees of freedom are more important in the P-wave part of nuclear potential energy than in the S-wave one.

There are a few other important issues worth discussing. First, the calculations must be self-consistent in a sense that both G-matrix and effective nucleon potential, entering the BG equation should be determined at the same order from the system of coupled equations since effective nucleon potential is itself determined by the G-matrix. This is not done yet. Second remark concerns the problem of removing of the off-shell ambiguities. EFT offers, in principle, the potential possibility to avoid this difficulty order by order. In practice, however, it may turn out that the cancelation of the unwanted contributions can be achieved only if all graphs, relevant at a given chiral order are included. It may turn out very complicated technical task. Third, the nuclear dressing of pions is to be considered. In the other words, pion self-energy must be computed to the order dictated by the counting rules.

In conclusion let us list the main ingredients which have to be included in any consistent nuclear EFT.

1. The local interaction terms as well as pion degrees of freedom must be included, probably up to NLO in the S, P and D-waves.
2. Nucleon and pion self-energies should be computed at a given chiral order.
3. To explain the saturation the three-nucleon forces seem to be needed.
4. The relations between the in-medium effective couplings and those arising in the vacuum 2-body and few-body calculations have to be established.
5. The reasonable way to generalize the nuclear matter EFT to treat the finite nuclei should be found.
6. All that must be supplied with the consistent formulation of the in-medium counting rules. This is probably a crucial issue for future studies.

As seen from this list we are at the beginning of the road. Many things must be done to formulate the entirely consistent and yet practical approach. However, the results already obtained on this way give a good reason to hope on a further progress in applying of the EFT methods to nuclear physics.

## REFERENCES

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- [1] A. Manohar, *Effective Field Theories*, [hep-ph/9506222](#); D. B. Kaplan, *Effective Field Theories*, [nucl-th/9506035](#).
- [2] S. Weinberg, Nucl. Phys. **B363**, (1991) 3.
- [3] D. B. Kaplan, M. Savage, and M. B. Wise, Nucl. Phys., **B534**, (1998) 329
- [4] M. C. Birse, J. A. McGovern and K. G. Richardson, Phys. Lett., **B464**, (1999) 169.
- [5] B. V. Krippa, [nucl-th/0002019](#);
- [6] H.-W. Hammer and R. Furnstahl, [nucl-th/0004043](#) .
- [7] R. Furnstahl, B. Serot and H.-B. Tang, Nucl. Phys., **A618**, (1997) 445.
- [8] M. Lutz, Nucl. Phys., **A642**, (1998) 171.
- [9] A. Bulgac, G . A. Miller and M. Strikman, Phys.Rev. **C56**, (1997) 3307.
- [10] M.Jaminon and C. Mahaux, Phys.Rev. **C40**, (1989) 354.
- [11] S. Beane, T. D. Cohen and D. Phillips, Nucl. Phys., **A632**, (1997) 445.
- [12] B. V. Krippa and B. L. G. Bakker, [hep-ph/0004178](#);
- [13] J. Gegelia Phys. Lett., **B429**, (1998) 227.
- [14] A. B. Migdal, Sov. Phys. JETP 10 (1960) 176.
- [15] H. Georgi and A. Manohar, Nucl. Phys., **B234**, (1984) 189.
- [16] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill, New-York, 1991)
- [17] C.J. Pethick and D. G. Ravenhall, Annu. Rev. Nucl. Part. Sci. **45** (1995) 429.
- [18] A. C. Muller and B. M. Sherill, Annu. Rev. Nucl. Part. Sci. **43** (1993) 529.
- [19] R. F. Bishop, Theor. Chim. Acta **80** (1991) 95.
- [20] W. H. Dickhoff and H. Muther, Rep. Prog. Phys. **55** (1992) 1947
- [21] J. Hormuzdiar and S. Hsu, [nucl-th/9811017](#)
- [22] T. Papenbrock and G. F. Bertsch, Phys.Rev. **C59**, (1999) 2052.
- [23] J. Polchinski, in Proc. of the 1992 TASI, eds J. Harvey and J. Polchinski (World Scientific, Singapore 1993).
- [24] M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett **B422**, (1998) 247; R.Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, (1998) 53.
- [25] S.Pepin, M. C. Birse and J. McGovern, [hep-ph/9912475](#)
- [26] J. Friar, Few-Body Systems **22**, (1997) 161
- [27] J.Carlson and R. Chiavilla, Rev. Mod. Phys. **70** (1998) 743
- [28] T. H. R. Skyrme, Nucl. Phys., **9**, (1959) 615.
- [29] R. Furnstahl and J. C. Hackworth, Phys.Rev. **C56** (1997) 2875.
- [30] D. Vautherin and D. Brink, Phys.Rev. **C5**, (1972) 629.
- [31] B. Lynn, Nucl. Phys, **B402**, (1993) 281